Intonation Exercises

Four Part Chorales

In Just Intonation

By

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Chapter 1

The 5-Limit Major Scale

I. Introduction

In just intonation, one way to construct the C major scale is to stack a series of perfect fifths (and reduce by octaves as necessary), starting on the fourth scale degree. This is the Pythagorean Major Scale, and this is the starting point for the scales used in Mixed Sagittal notation. The resulting scale's tuning properties are:

Note	Integer Ratio	Factors	Deviation Away From 12-E.T. in Cents (Rounded)
C	1:1	1	0
D	9:8	$2^{-3},3^{2}$	+4
E	81:64	$2^{-6},3^{4}$	+8
F	4:3	$2^{2},3^{-1}$	-2
G	3:2	$2^{-1},3$	+2
A	27:16	$2^{-4},3^{3}$	+6
B	243:128	$2^{-7},3^{5}$	+10

The factors column will provide simplification as the ratios become more complex later in this book.

Another way to construct the C major scale is to take the notes from the perfectly tuned I, IV, and V chords, the Primary Triads. This scale was first described by the Roman philosopher and scientist, Claudius Ptolemaeus, or more commonly, Ptolemy. Microtonal pioneer Harry Partch called this the “Ptolemaic Sequence.” It is also called the 5-limit Major Scale for it's mathematical properties; whereas, the Pythagorean Major Scale is 3-limit. Instead of stacking perfect fifths, the Primary chords are spelled with one perfect fifth (3:2) and one simple just major third (5:4):

Note	Integer Ratio	Factors	Deviation in Cents
C	1:1	1	0
D	9:8	$2^{-3},3^{2}$	+4
E	5:4	$2^{-2},5$	-14
F	4:3	$2^{2},3^{-1}$	-2
G	3:2	$2^{-1},3$	+2
A	5:3	$3^{-1},5$	-16
B	15:8	$2^{-4},3,5$	-12

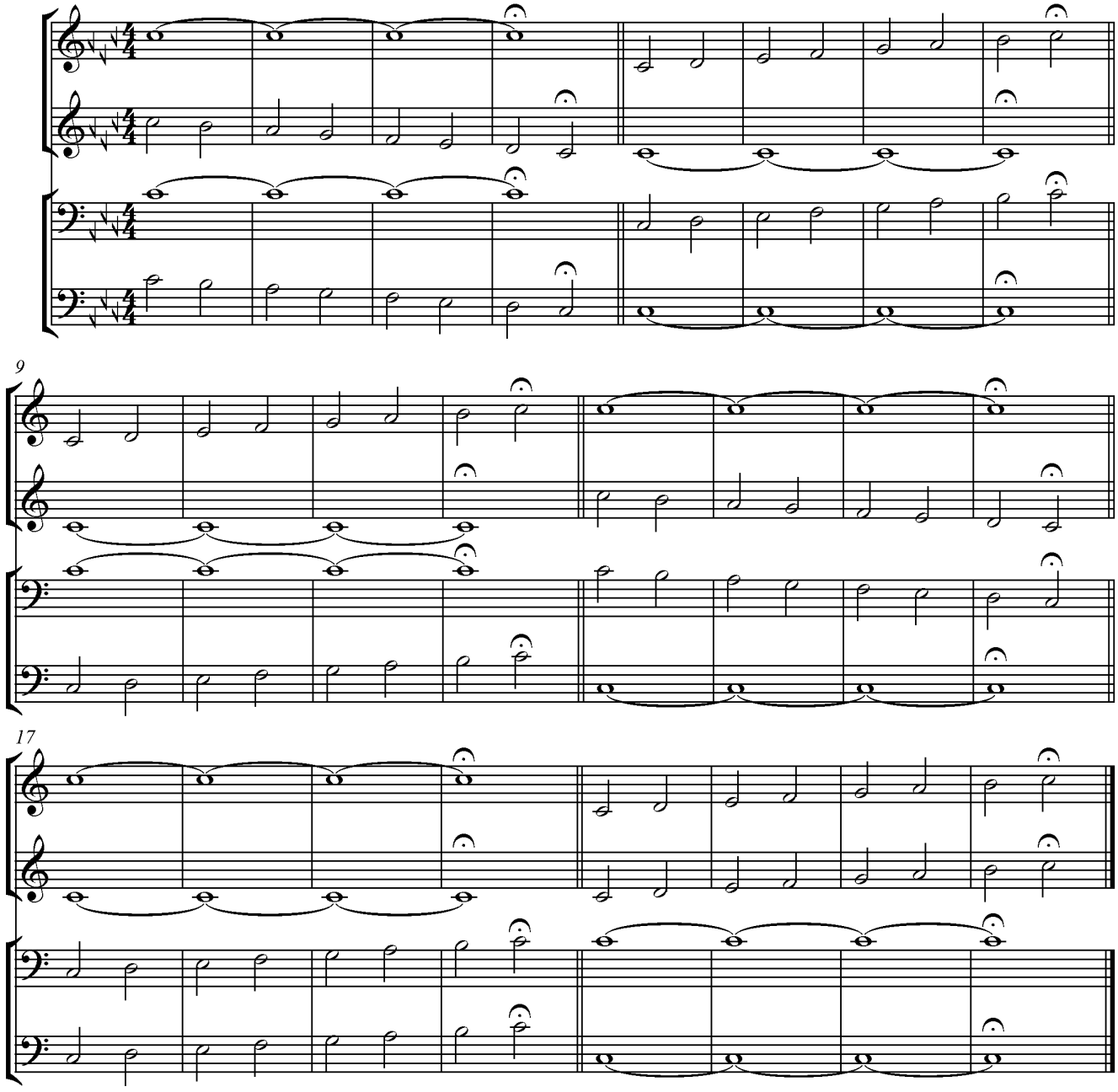
These are subtly different scales, but the differences can be functionally described with what is called a Syntonic Comma. Mixed Sagittal's way of notating this important interval are these symbols:

Symbol	Integer Ratio	Factors	Deviation in Cents
𝅘𝅥	81:80	$2^{-4},3^{4},5^{-1}$	+22
𝅘𝅥	80:81	$2^{4},3^{-4},5$	-22

More about the Syntonic Comma is in Chapter 2; here, simply note that this accidental is added as part of the key signature to specify that in C major that the E, A, and B are lowered by this amount compared to the Pythagorean Major Scale, to form the most resonant, consonant harmony.

The following chorales are intended to familiarize students with the 5-Limit Major Scale.

1Ia



II. Sharps and Flats

Sharps and flats in Mixed Sagittal are used very similarly to how the sharps and flats that every musician who plays Western music is familiar with. Like the syntonic commas above, they are also regular accidentals and all the normal notational rules apply. The extent to which they affect pitch is slightly different:

Symbol	Ratio	Factors	Adjustment (cents)
♯	2187:2048	$2^{-11},3^{7}$	+114
♭	2048:2187	$2^{11},3^{-7}$	-114

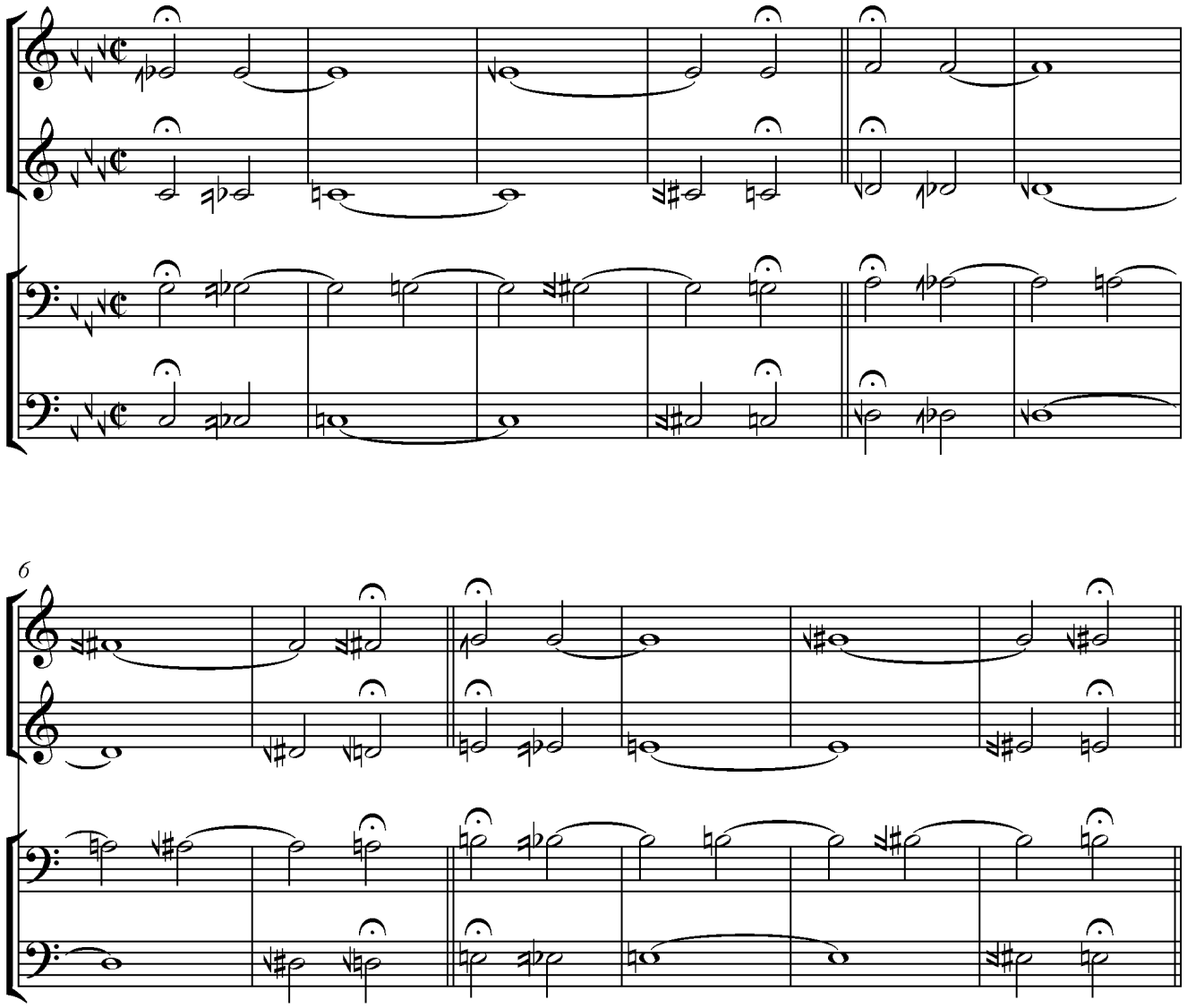
Furthermore, when combined with syntonic commas, the pitch adjustments are additive. Thus, if you see a 𝅘𝅥♯ in front of a note, then the pitch adjustment would be -22 +114 = +92 cents, with rounding errors added from how the cents are calculated compared to the ratios.

Symbol	Ratio	Factors	Adjustment (cents)
𝅘𝅥♭	163840:177147	$2^{14},3^{-11},5$	-135
𝅥♭	128:135	$2^{7},3^{-3},5^{-1}$	-92
𝅘𝅥♯	135:128	$2^{-7},3^{3},5$	+92
𝅥♯	177147:163840	$2^{-14},3^{11},5^{-1}$	+135
𝅥𝅥♭	24:25	$2^{3},3,5^{-2}$	-71
𝅘𝅥𝅥♯	25:24	$2^{-3},3^{-1},5^{2}$	+71

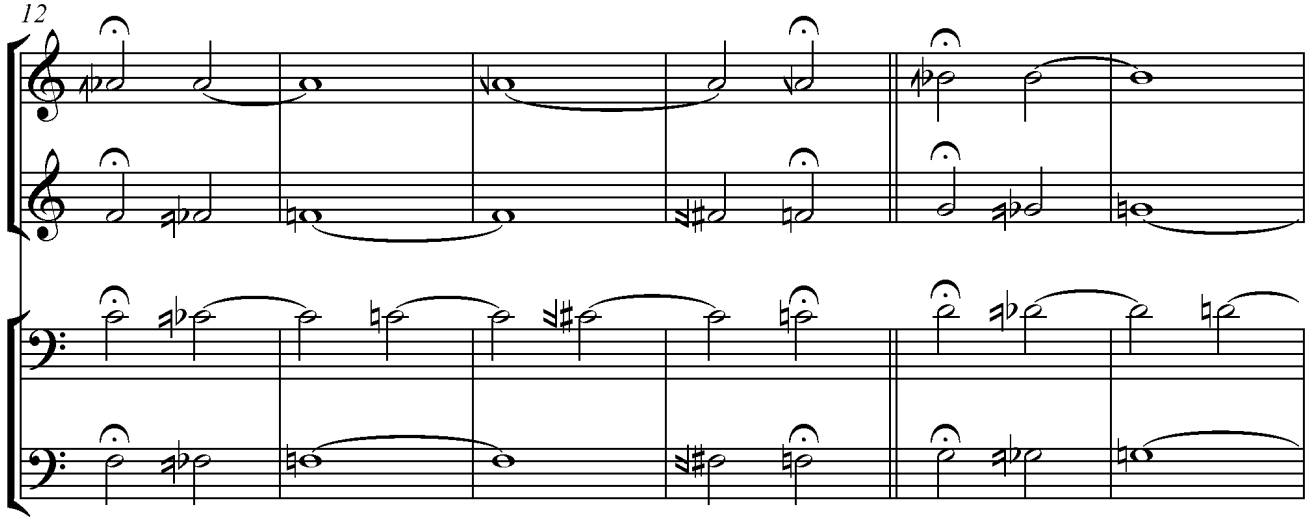
The 𝅥𝅥♯ and 𝅘𝅥𝅥♭ symbols represent two syntomic commas stacked in the interval.

The chorales on the following pages are drills to familiarize musicians with these intonation tools. As with all chorales in this book, the goal is to find consonant, beautiful harmony. The astute observer should begin to see why “adjusting on the fly” in traditional music becomes so necessary; as these drills progress, virtually every possible note will have these accidentals applied differently at least once, showing how different harmonic contexts require different tunings to achieve just, consonant harmony.

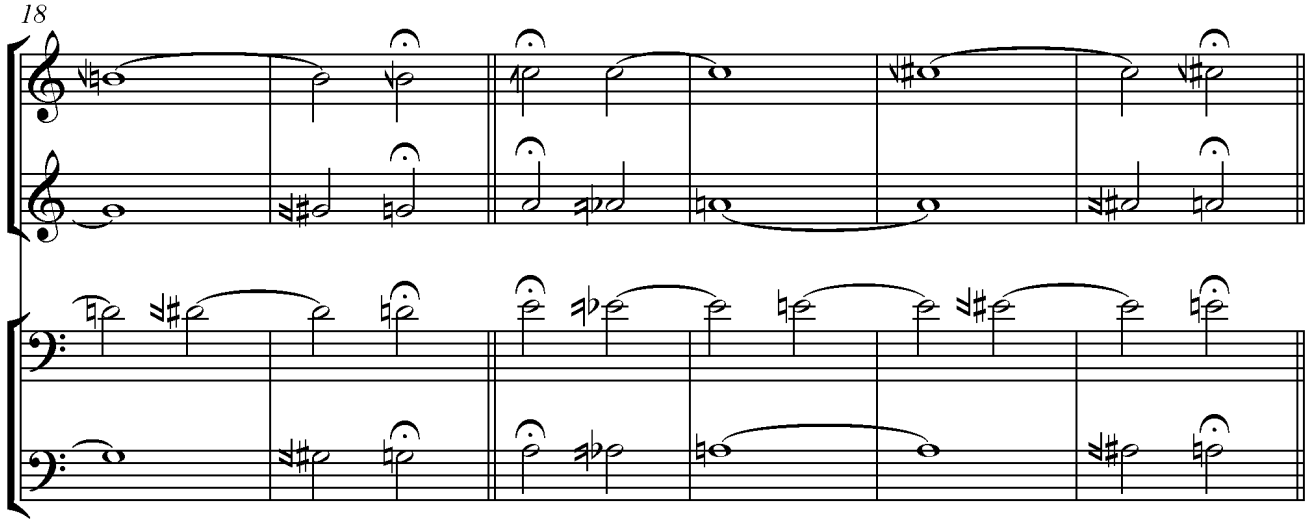
2.II.d



12



18



24